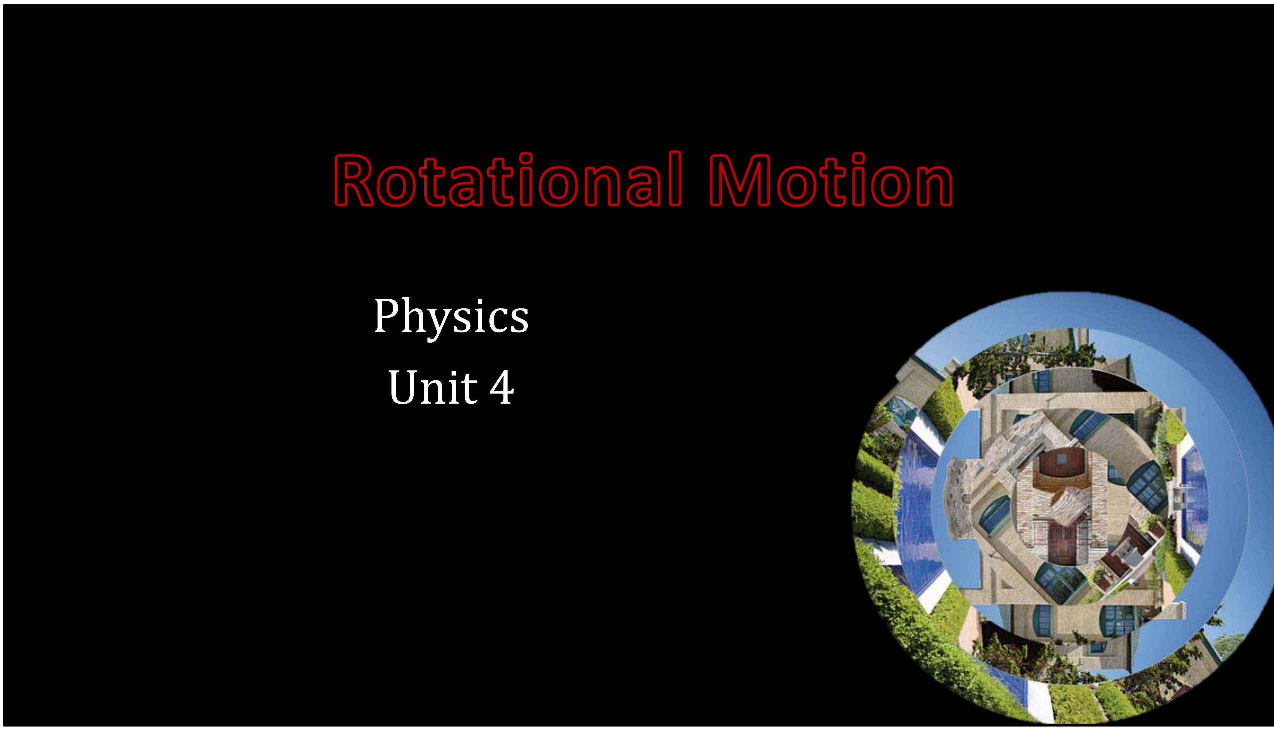


Physics Unit 4

- This Slideshow was developed to accompany the textbook
    - *OpenStax Physics*
        - *Available for free at https://openstaxcollege.org/textbooks/college-physics*
    - *By OpenStax College and Rice University*
    - *2013 edition*
- Some examples and diagrams are taken from the textbook.

Slides created by
Richard Wright, Andrews Academy
rwright@andrews.edu

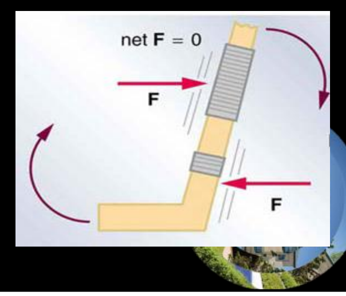
In this lesson you will…

- State the first condition of equilibrium.
- Explain static equilibrium.
- Explain dynamic equilibrium.
- State the second condition that is necessary to achieve equilibrium.
- Explain torque and the factors on which it depends.
- Describe the role of torque in rotational mechanics.

04-01 EQUILIBRIUM

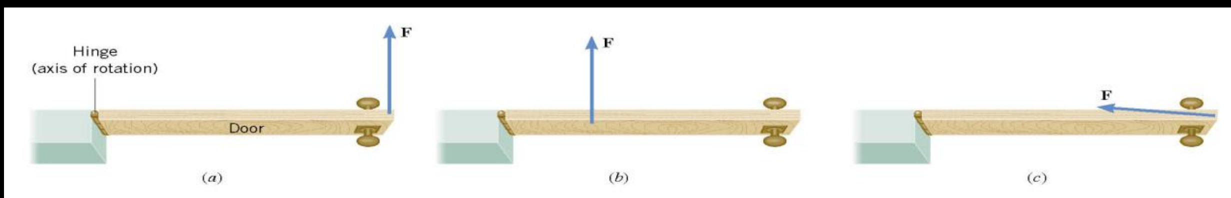
04-01 Equilibrium

- Statics
  - Study of forces in equilibrium
  - Equilibrium means no acceleration
- First condition of equilibrium
  - $net\ F = 0$
    - $F_x = 0$ and $F_y = 0$
    - They can still rotate, so…



04-01 Equilibrium

- Think of opening a door

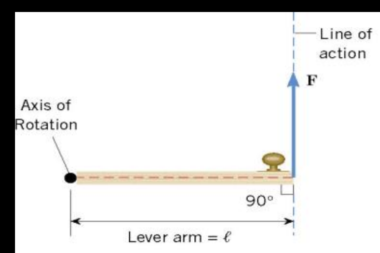


- Which opens the door the best?
  - Picture a
- Big force → large torque
- Force away from pivot → large torque
- Force directed ⊥ to door → large torque

04-01 Equilibrium

- $\tau = F \times r$
  - This means we use the component of the force that is perpendicular to the lever arm
- $\tau = F_{\perp}\, r$
- $\tau = F\, r \sin \theta$
  - θ is the angle between the force and the radius

- Unit: Nm
  - CCW → +
  - CW → -



04-01 Equilibrium

- You are meeting the parents of your new “special” friend for the first time. After being at their house for a couple of hours, you walk out to discover the little brother has let all the air out of one of your tires. Not knowing the reason for the flat tire, you decide to change it. You have a 50-cm long lug-wrench attached to a lugnut as shown. If 900 Nm of torque is needed, how much force is needed?

- F = 2078 N
- Less force required if pushed at 90°

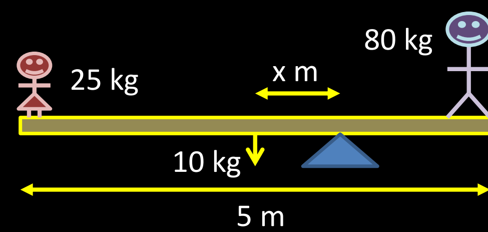
$$\tau = Fr\sin\theta$$
$$900\ Nm = F(0.5m)(\sin 120°)$$
$$F = 2078\ N$$

04-01 Equilibrium

- Second condition of equilibrium
  - Net torque = 0

04-01 Equilibrium

- A 5 m, 10 kg seesaw is balanced by a little girl (25 kg) and her father (80 kg) at opposite ends as shown below. How far from the seesaw's center of mass must the fulcrum be placed?
  - 1.20 m
- How much force must the fulcrum support?
  - 1029 N



$$\sum\tau = 0$$

$$(25\,kg \cdot 9.8\,m/s^2)(2.5\,m + x) + (10\,kg \cdot 9.8\,m/s^2)x$$
$$- (80\,kg \cdot 9.8\,m/s^2)(2.5\,m - x) = 0$$

$$612.5\,Nm + 245\,N\,x + 98\,N\,x - 1960\,Nm + 784\,N\,x = 0$$

$$-1347.5\,Nm + 1127\,N\,x = 0$$

$$1127\,N\,x = 1347.5\,Nm$$

$$x = 1.20\,m$$

$$\sum F = -W_g - W_f + F_f = 0$$

$$-(25\,kg)\left(9.8\frac{m}{s^2}\right) - (80\,kg)\left(9.8\frac{m}{s^2}\right) + F_f = 0$$

$$-1029\,N + F_f = 0$$

$$F_f = 1029\,N$$

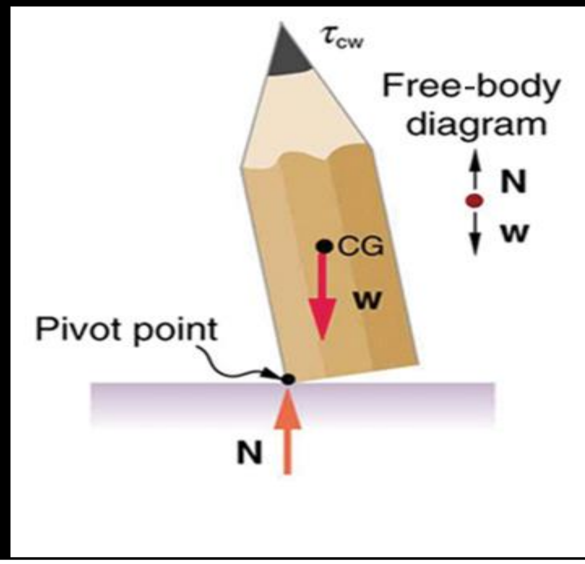
04-01 Homework

- Twist out the answers to these torque questions
- Read 9.3, 9.4


In this lesson you will…
• State the types of equilibrium.
• Describe stable and unstable equilibriums.
• Describe neutral equilibrium.
• Discuss the applications of Statics in real life.
• State and discuss various problem-solving strategies in Statics.
04-02 STABILITY AND APPLICATIONS

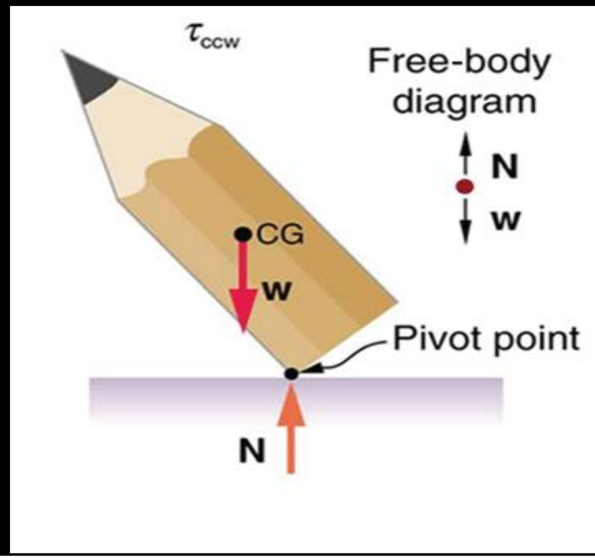
04-02 Stability and Applications

- Stable equilibrium
  - When displaced from equilibrium, the system experiences a net force or torque in a direction opposite to the direction of the displacement.



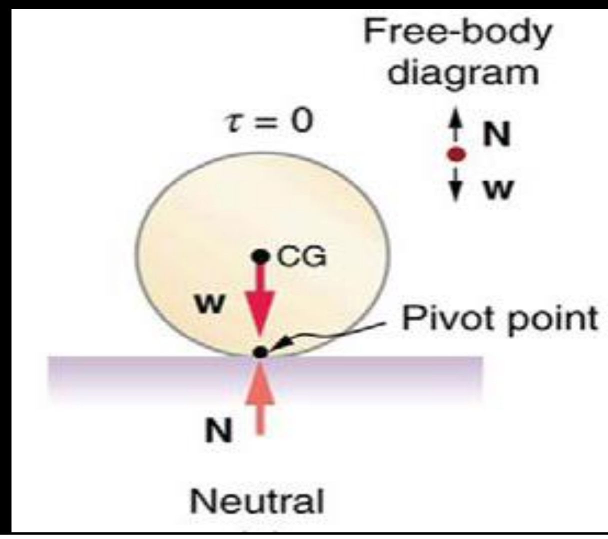
04-02 Stability and Applications

- Unstable equilibrium
  - When displaced from equilibrium, the net force or torque is in same direction of the displacement



04-02 Stability and Applications

- Neutral Equilibrium
  - Equilibrium is independent of displacement from its original position



04-02 Stability and Applications

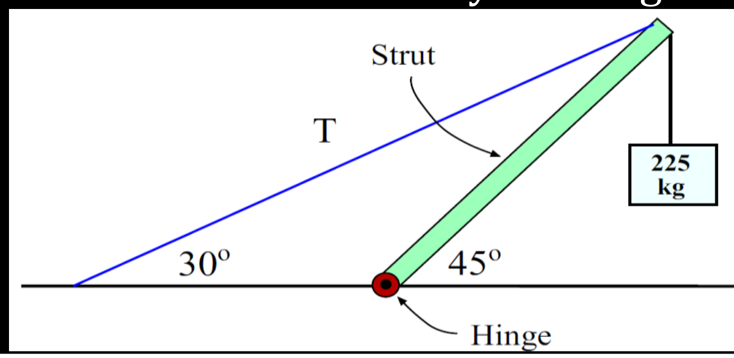
Problem-Solving Strategy for Static Equilibrium

1. Is it in equilibrium? (no acceleration or accelerated rotation)
2. Draw free body diagram
3. Apply $\sum F = 0$ and/or $\sum \tau = 0$
   a. Choose a pivot point to simplify the problem
4. Check your solution for reasonableness.

04-02 Stability and Applications

- The system is in equilibrium. A mass of 225 kg hangs from the end of the uniform strut whose mass is 45.0 kg. Find (a) the tension *T* in the cable and the (b) horizontal and (c) vertical force components exerted on the strut by the hinge.

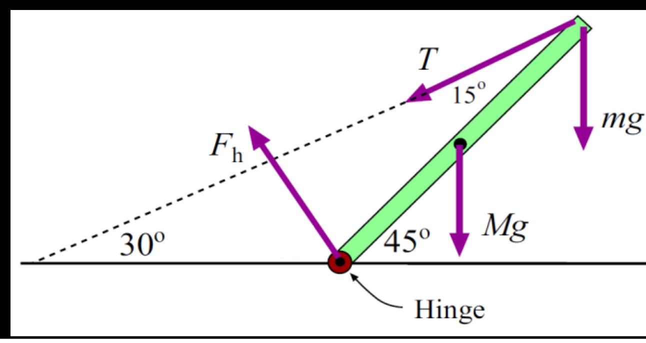
- Free body diagram next slide



04-02 Stability and Applications

- $m = 225\ kg, M = 45.0\ kg, T = ?, F_{hx} = ?, F_{hy} = ?$

- $T = 6627\ N$
- $F_{hx} = 5739\ N$
- $F_{hy} = 5959\ N$



Use hinge as pivot point to eliminate need for F_h

$$\sum \tau = 0$$

$$F_h(0) + T(L)\sin 15° - (Mg)\left(\frac{L}{2}\right)\sin 45° - mg(L)\sin 45° = 0$$

$$0 + TL\sin 15° - (45\ kg)\left(9.8\frac{\text{m}}{\text{s}^2}\right)\left(\frac{\text{L}}{2}\right)\sin 45° - (225\ kg)\left(9.8\frac{m}{s^2}\right)(L)\sin 45° = 0$$

$$TL\sin 15° - 155.917\ N\ L - 1559.170\ N\ L = 0$$

$$TL\sin 15° = 1715.087\ N\ L$$

$$T = \frac{1715.087\ N}{\sin 15°} = 6627\ N$$

Find F_{hx}

$$\sum F_x = 0$$

$$F_{hx} - T\cos 30° + 0 + 0 = 0$$

$$F_{hx} = 6627\ N\cos 30° = 5739\ N$$

Find F_{hy}

$$\sum F_y = 0$$

$$F_{hy} - T\sin 30° - Mg - mg = 0$$

$$F_{hy} = T\sin 30° + Mg + mg$$

$$F_{hy} = 6627\ N\sin 30° + (45\ kg)\left(9.8\frac{m}{s^2}\right) + (225\ kg)\left(9.8\frac{m}{s^2}\right) = 5959\ N$$

04-02 Homework

- Pick a stable position while you apply your knowledge.
- Read 9.5, 9.6

In this lesson you will…

- Describe different simple machines.
- Calculate the mechanical advantage.
- Explain the forces exerted by muscles.
- State how a bad posture causes back strain.
- Discuss the benefits of skeletal muscles attached close to joints.
- Discuss various complexities in the real system of muscles, bones, and joints.

04-03 SIMPLE MACHINES, MUSCLES, AND JOINTS

04-03 Simple Machines, Muscles, and Joints

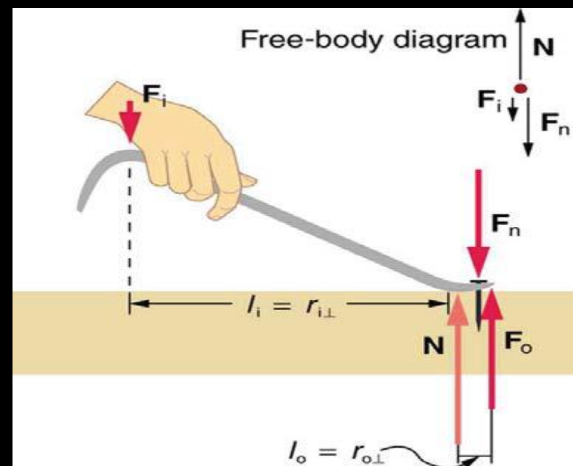
- Machines make work easier
  - Energy is conserved so same amount of energy with or without machine

- Mechanical Advantage (MA)
  - $MA = \frac{F_o}{F_i}$

$F_o = force\ out$
$F_i = force\ in$

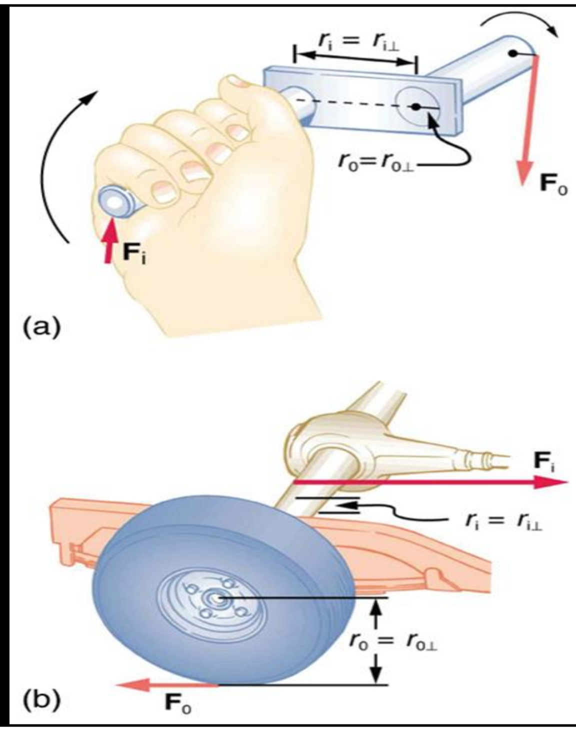
04-03 Simple Machines, Muscles, and Joints

- Lever
- Uses torques with pivot at N
- $F_i l_i = F_o l_o$
- $\frac{F_o}{F_i} = \frac{l_i}{l_o}$
- When F ↑, l ↓
- $MA = \frac{F_o}{F_i} = \frac{d_i}{d_o}$



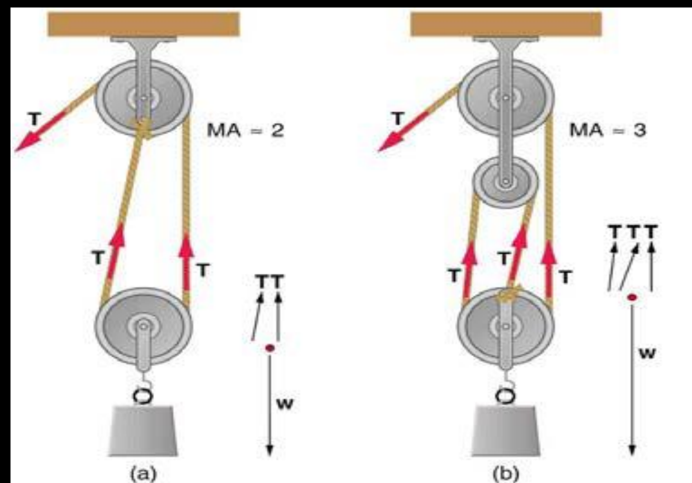
04-03 Simple Machines,

- Other simple machines
- Wheel and Axle
  - Lever
- Inclined Plane
  - Ramp – less force to slide up, but longer distance
- Screw
  - Inclined plane wrapped around a shaft
- Wedge
  - Two inclined planes



04-03 Simple Machines, Muscles, and Joints

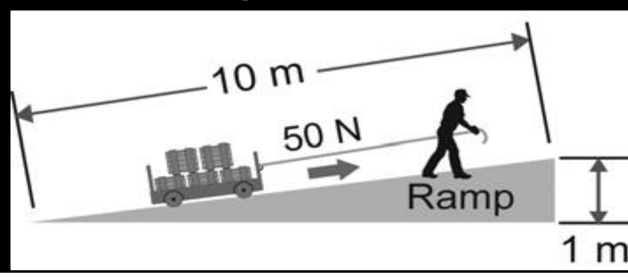
- Pulley
  - Grooved wheel
  - Changed direction of force
  - In combination, can decrease force



04-03 Simple Machines, Muscles, and Joints

- What is the mechanical advantage of the inclined plane?
- MA = 10

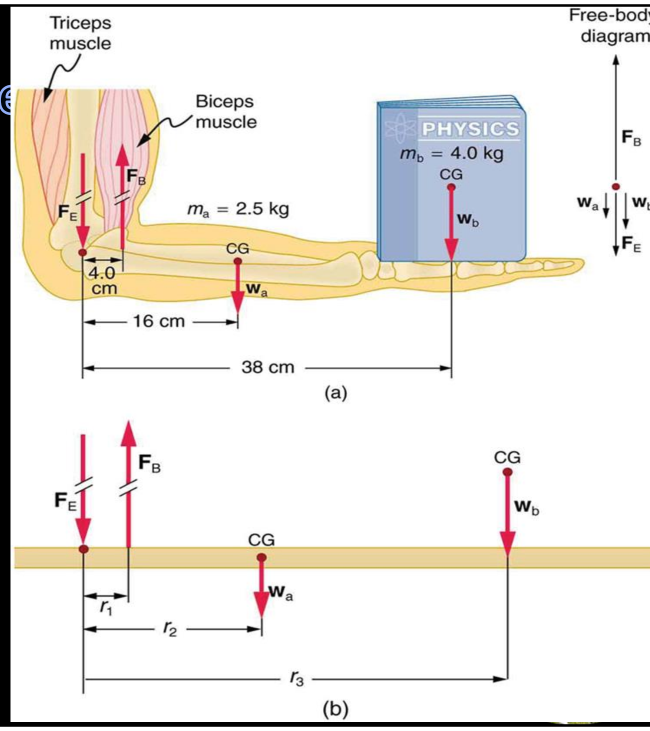
- What is the weight of the cart assuming no friction?
- $F_o = 500$ N



$$MA = \frac{F_o}{F_i} = \frac{d_i}{d_o}$$
$$MA = \frac{10\ m}{1\ m} = 10$$
$$\frac{F_o}{50\ N} = \frac{10\ m}{1\ m}$$
$$F_o = 500\ N$$

04-03 Simple Machine

- Muscles only contract, so they come in pairs
- Muscles are attached to bones close to the joints using tendons
- This makes the muscles supply larger force than is lifted
- Input force > output force
- MA < 1



Huge forces can be created this way.


SPAR
100m M
Rd 1 Heat 4
ER 9.86
CR 9.99
0.0

04-03 Homework

- Machines can't help you much here, exercise your mental muscles instead
- Read 10.1, 10.2

In this lesson you will…

- Describe uniform circular motion.
- Explain non-uniform circular motion.
- Calculate angular acceleration of an object.
- Observe the link between linear and angular acceleration.
- Observe the kinematics of rotational motion.
- Derive rotational kinematic equations.
- Evaluate problem solving strategies for rotational kinematics.

04-04 KINEMATICS OF ROTATIONAL MOTION

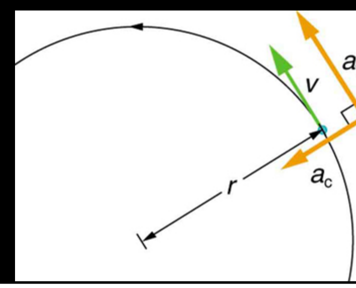
04-04 Kinematics of Rotational Motion

- Rotational motion
- Describes spinning motion
- θ is like x
  - $x = r\theta$ → position
- ω is like v
  - $\omega = \frac{\Delta\theta}{\Delta t}$
  - $v = r\omega$ → velocity
- α is like a
  - $\alpha = \frac{\Delta\omega}{\Delta t}$
  - $a_t = r\alpha$ → acceleration

CCW is +
CW is -

04-04 Kinematics of Rotational Motion

- Two components to acceleration
  - Centripetal
    - Toward center
    - Changes direction only since perpendicular to v
    - $a_c = \frac{v^2}{r}$
- Tangental (linear)
  - Tangent to circle
  - Changes speed only since parallel to v
  - $a_t = r\alpha$



04-04 Kinematics of Rotational Motion

- Equations of kinematics for rotational motion are same as for linear motion

- $\theta = \overline{\omega} t$
- $\omega = \omega_0 + \alpha t$
- $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
- $\omega^2 = \omega_0^2 + 2\alpha\theta$

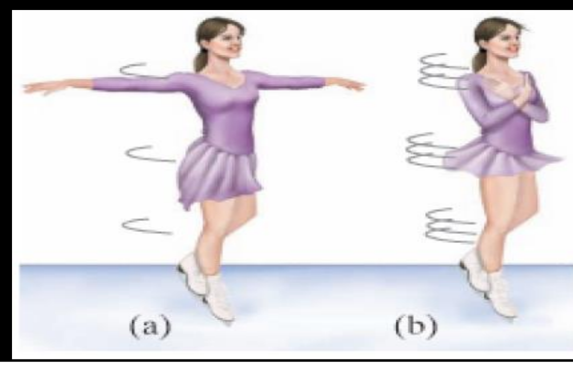
04-04 Kinematics of Rotational Motion

- Reasoning Strategy

1. Examine the situation to determine if rotational motion involved
2. Identify the unknowns (a drawing can be useful)
3. Identify the knowns
4. Pick the appropriate equation based on the knowns/unknowns
5. Substitute the values into the equation and solve
6. Check to see if your answer is reasonable

04-04 Kinematics of Rotational Motion

- A figure skater is spinning at 0.5 rev/s and then pulls her arms in and increases her speed to 10 rev/s in 1.5 s. What was her angular acceleration?
- 39.8 rad/s^2



$$\omega_0 = 0.5\frac{rev}{s}\left(\frac{2\pi\, rad}{rev}\right) = \pi\frac{rad}{s}$$

$$\omega = 10\frac{rev}{s}\left(\frac{(2\pi\, rad)}{rev}\right) = 20\pi\frac{rad}{s}$$

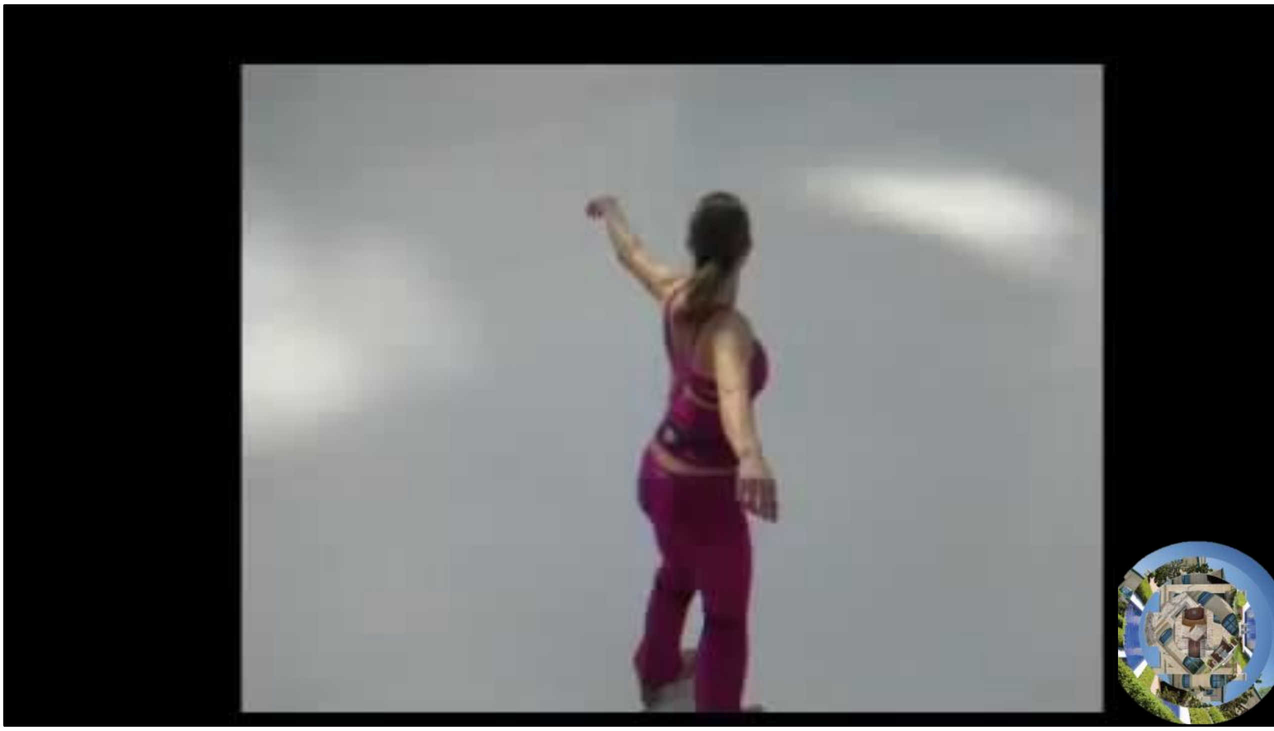
$$t = 1.5\ s$$

$$\omega = \omega_0 + \alpha t$$

$$20\pi\frac{rad}{s} = \pi\frac{rad}{s} + \alpha(1.5\ s)$$

$$19\pi\frac{rad}{s} = \alpha(1.5\ s)$$

$$\alpha = 39.8\frac{rad}{s^2}$$



04-04 Kinem  tional Motion

- A ceiling fan has 4 evenly spaced blades of negligible width. As you are putting on your shirt, you raise your hand. It brushes a blade and then is hit by the next blade. If the blades were rotating at 4 rev/s and stops in 0.01 s as it hits your hand, what angular displacement did the fan move after it hit your hand?
- $\theta = 0.02 \text{ rev} = 0.126 \text{ rad} = 7.2°$

$$\omega_0 = 4\frac{rev}{s}, t = 0.01\, s$$

$$\theta = \overline{\omega} t$$

$$\theta = \left(\frac{\omega + \omega_0}{2}\right) t$$

$$\theta = \left(\frac{0 + 4\frac{rev}{s}}{2}\right)(0.01\, s) = 0.02\, rev = 0.126\, rad = 7.2°$$

04-04 Homework

- Spin up your mind and toss out some answers
- Read 10.3, 10.4

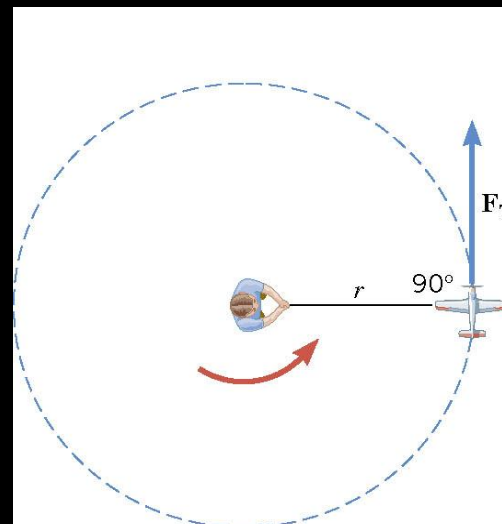
In this lesson you will…

- Understand the relationship between force, mass and acceleration.
- Study the turning effect of force.
- Study the analogy between force and torque, mass and moment of inertia, and linear acceleration and angular acceleration.
- Derive the equation for rotational work.
- Calculate rotational kinetic energy.
- Demonstrate the Law of Conservation of Energy.

04-05 DYNAMICS OF ROTATIONAL MOTION

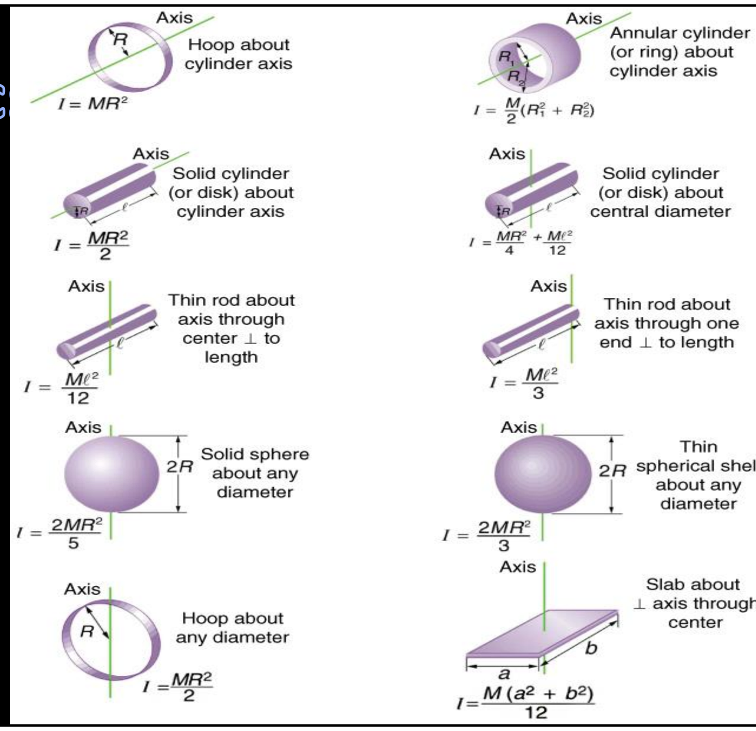
04-05 Dynamics of Rotational Motion

- $\tau = F_T r$
  - $F_T = m a_t$
- $\tau = m a_t r$
  - $a_t = r\alpha$
- $\tau = m r^2 \alpha$
  - $I = m r^2$ → Moment of inertia of a particle
- $\tau = I\alpha$
  - Newton's second law for rotation
  - α is in rad/s^2



04-05 Dynamic

- Moment of Inertia (I) measures how much an object wants to keep rotating (or not start rotating)
- Use calculus to find $I = \sum mr^2$
- Unit:
  - kg m^2
- Page 328 lists I for many different mass distributions



04-05 Dynamics of Rotational Motion

- Work for rotation
  - $W = F\Delta s$
  - $W = Fr\frac{\Delta s}{r}$
  - $W = \tau\theta$
- Kinetic Energy
  - $KE_{rot} = \frac{1}{2}I\omega^2$
- Conservation of Mechanical Energy
  - $PE_i + KE_i = PE_f + KE_f$
  - Remember that the KE can include both translational and rotational.

$$KE = \frac{1}{2}mv^2$$

I is the same as *m* for rotation

Conservation of ME is when closed system

04-05 Dynamics of Rotational Motion

- Zorch, an archenemy of Superman, decides to slow Earth's rotation to once per 28.0 h by exerting an opposing force at and parallel to the equator. Superman is not immediately concerned, because he knows Zorch can only exert a force of 4.00×10^{7} N (a little greater than a Saturn V rocket's thrust). How long must Zorch push with this force to accomplish his goal? (This period gives Superman time to devote to other villains.)
- $1.26\times10^{11}\ yr$

$$F=-4\times10^{7}\ N, r=6.38\times10^{6}\ m, m_E=5.98\times10^{24}\ kg$$

$$\omega_0=\frac{1\ rev}{24\ h}\left(\frac{2\pi\ rad}{1\ rev}\right)\left(\frac{1\ h}{3600\ s}\right)=\frac{\pi}{43200}\frac{rad}{s}\approx 7.272\times10^{-5}\frac{rad}{s}$$

$$w_f=\frac{1\ rev}{28\ h}\left(\frac{2\pi}{1\ rev}\right)\left(\frac{1\ h}{3600\ s}\right)=\frac{\pi}{50400}\frac{rad}{s}\approx 6.233\times10^{-5}\frac{rad}{s}$$

$$\tau=I\alpha$$

$$\alpha=\frac{\Delta\omega}{\Delta t}=\frac{\omega_f-\omega_0}{t}$$

$$\tau=I\left(\frac{\omega_f-\omega_0}{t}\right)$$

$$t=\frac{I(\omega_f-\omega_0)}{\tau}$$

$$I=\frac{2}{5}MR^2, \tau=FR$$

$$t=\frac{\frac{2}{5}MR^{\cancel{2}}(\omega_f-\omega_0)}{F\cancel{R}}$$

$$t=\frac{2(5.98\times10^{24}\ kg)(6.38\times10^{6}\ m)\left(6.233\times10^{-5}\frac{rad}{s}-7.272\times10^{-5}\frac{rad}{s}\right)}{5(-4.00\times10^{7}\ N)}$$

$$t=3.964\times10^{18}\ s=1.101\times10^{15}\ h=4.588\times10^{13}\ d=1.26\times10^{11}\ yr$$

04-05 Dynamics of Rotational Motion

- A solid sphere (m = 2 kg and r = 0.25 m) and a thin spherical shell (m = 2 kg and r = 0.25 m) roll down a ramp that is 0.5 m high. What is the velocity of each sphere as it reaches the bottom of the ramp?

- Solid: 2.65 m/s
- Shell: 2.42 m/s

- Notice masses canceled so mass didn't matter

Solid Sphere: $m = 2\ kg, r = 0.25\ m, h_0 = 0.5\ m, h_f = 0, v_0 = 0, \omega_0 = 0$

$$PE_i + KE_{trans0} + KE_{rot0} = PE_f + KE_{transf} + KE_{rotf}$$

$$mgh_0 + 0 + 0 = 0 + \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

$$\cancel{m}gh_0 = \frac{1}{2}\cancel{m}v_f^2 + \frac{1}{2}\left(\frac{2}{5}\cancel{m}r^2\right)\omega_f^2$$

$$gh_0 = \frac{1}{2}v_f^2 + \frac{1}{5}r^2\left(\frac{v_f}{r}\right)^2$$

$$gh_0 = \frac{1}{2}v_f^2 + \frac{1}{5}v_f^2$$

$$gh_0 = \frac{7}{10}v_f^2$$

$$v_f = \sqrt{\frac{10}{7}gh_0}$$

$$v_f = \sqrt{\frac{10}{7}\left(9.80\frac{m}{s^2}\right)(0.5\ m)} = 2.65\frac{m}{s}$$

Shell: $m = 2\ kg, r = 0.25\ m, h_0 = 0.5\ m, h_f = 0, v_0 = 0, \omega_0 = 0$

$$PE_i + KE_{trans0} + KE_{rot0} = PE_f + KE_{transf} + KE_{rotf}$$

$$mgh_0 + 0 + 0 = 0 + \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

$$\cancel{m}gh_0 = \frac{1}{2}\cancel{m}v_f^2 + \frac{1}{2}\left(\frac{2}{3}\cancel{m}r^2\right)\omega_f^2$$

$$gh_0 = \frac{1}{2}v_f^2 + \frac{1}{3}r^2\left(\frac{v_f}{r}\right)^2$$

$$gh_0 = \frac{1}{2}v_f^2 + \frac{1}{3}v_f^2$$

$$gh_0 = \frac{5}{6}v_f^2$$

$$v_f = \sqrt{\frac{6}{5}gh_0}$$

$$v_f = \sqrt{\frac{6}{5}\left(9.80\frac{m}{s^2}\right)(0.5\,m)} = 2.42\frac{m}{s}$$

04-05 Homework

- Don't spin your wheels as you use energy to solve these problems
- Read 10.5

In this lesson you will…

- Understand the analogy between angular momentum and linear momentum.
- Observe the relationship between torque and angular momentum.
- Apply the law of conservation of angular momentum.

04-06 ANGULAR MOMENTUM

04-06 Angular Momentum

- Linear momentum
  - p = mv
- Angular momentum
  - L = Iω
  - Unit:
  - kg m^2/s
  - ω must be in rad/s
- When you rotate something you exert a torque.
- More torque = faster change in angular momentum
- $\tau_{net} = \frac{\Delta L}{\Delta t}$
- Like $F = \frac{\Delta p}{\Delta t}$

Demo with the rotation rods

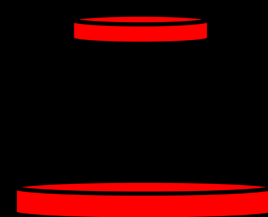
I is like *m* for rotational motion

04-06 Angular Momentum

- Linear momentum of a system is conserved if $F_{net} = 0$
  - $p_0 = p_f$
- Angular momentum of a system is also conserved if $\tau_{net} = 0$
  - $L_0 = L_f$

04-06 Angular Momentum

- A 10-kg solid disk with r = 0.40 m is spinning at 8 rad/s. A 9-kg solid disk with r = 0.30 m is dropped onto the first disk. If the first disk was initially not rotating, what is the angular velocity after the disks are together?
  - ω = 5.31 rad/s
- What was the torque applied by the first disk onto the second if the collision takes 0.01 s?
  - τ = 215 Nm



Disk 1:

$$\omega_0 = 8\frac{rad}{s}, r = 0.4\, m, m = 10\, kg$$

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(10\, kg)(0.4\, m)^2 = 0.8\, kg \cdot m^2$$

Disk 2:

$$\omega_0 = 0\frac{rad}{s}, r = 0.3\, m, m = 9\, kg$$

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(9\, kg)(0.3\, m)^2 = 0.405\, kg\, m^2$$

$$L = I\omega$$

$$L_0 = L_f$$

$$(0.8\, kgm^2)\left(8\frac{rad}{s}\right) + (0.405\, kgm^2)(0) = (0.8\, kgm^2 + 0.405\, kgm^2)\omega$$

$$6.4\, kgm^2 = (1.205\, kgm^2)\omega$$

$$\omega = 5.31\, rad/s$$

$$\tau = \frac{\Delta L}{\Delta t}$$

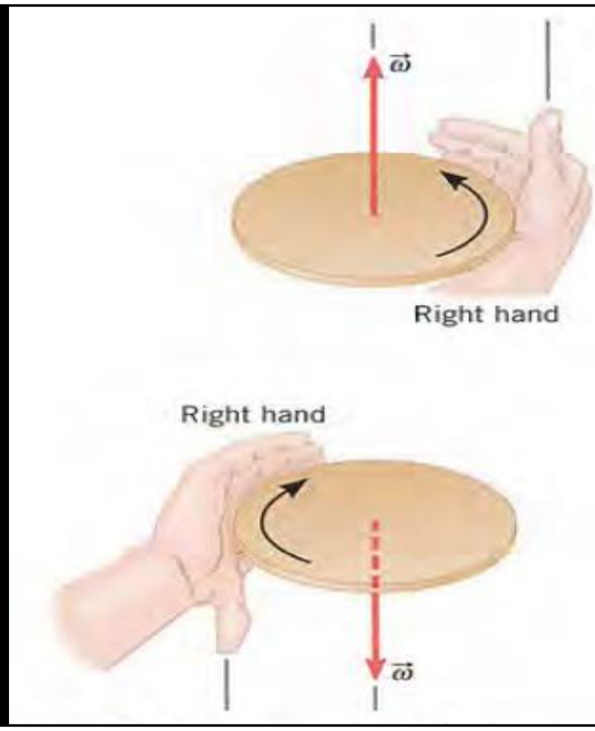
$$\tau = \frac{0.405\, kgm^2\left(5.31\frac{rad}{s}\right) - 0}{0.01\, s} = 215.055\, Nm$$

04-06 Angular Momentum

- Angular Momentum conserved if net external torque is zero
- Linear Momentum conserved if net external force is zero
- Kinetic Energy conserved if elastic collision

04-06 Angular

- Direction of angular quantities
  - Right-hand Rule
  - Hold hand out with thumb out along axis
  - Curl your fingers in direction of motion (you may have to turn your hand upside down)
  - vector in direction of thumb



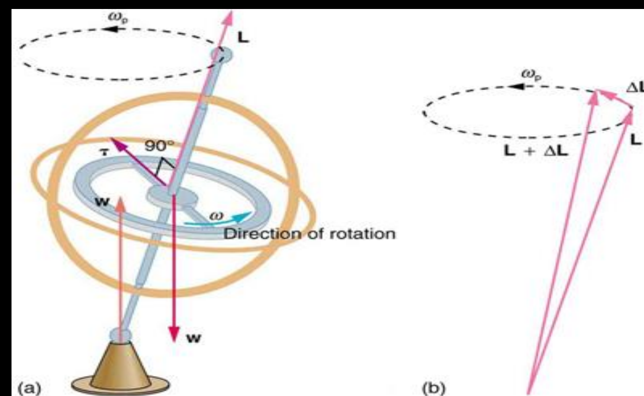
04-06 Angular Momentum

- A person is holding a spinning bicycle wheel while he stands on a stationary frictionless turntable. What will happen if he suddenly flips the bicycle wheel over so that it is spinning in the opposite direction?

Consider the system of turntable, person, and wheel. The total angular momentum before is **L** upward. Afterward, the total angular momentum must be the same. If the wheel is upside down, its angular momentum is **–L** so the angular momentum of the person must be **+2L.** So the person rotates in the direction the wheel was initially spinning.

04-06 Angular Momentum

- Gyroscopes
- **T**wo forces acting on a spinning gyroscope. The torque produced is perpendicular to the angular momentum, thus the direction of the torque is changed, but not its magnitude. The gyroscope *precesses* around a vertical axis, since the torque is always horizontal and perpendicular to **L** .
- If the gyroscope is ***not*** spinning, it acquires angular momentum in the direction of the torque (**L** = Δ**L**), and it rotates around a horizontal axis, falling over just as we would expect.



Use right-hand rule to find direction of torque.

Earth itself acts like a gigantic gyroscope. Its angular momentum is along its axis and points at Polaris, the North Star. But Earth is slowly precessing (once in about 26,000 years) due to the torque of the Sun and the Moon on its nonspherical shape.

04-06 Homework

- Let you momentum carry you through these problems